1.
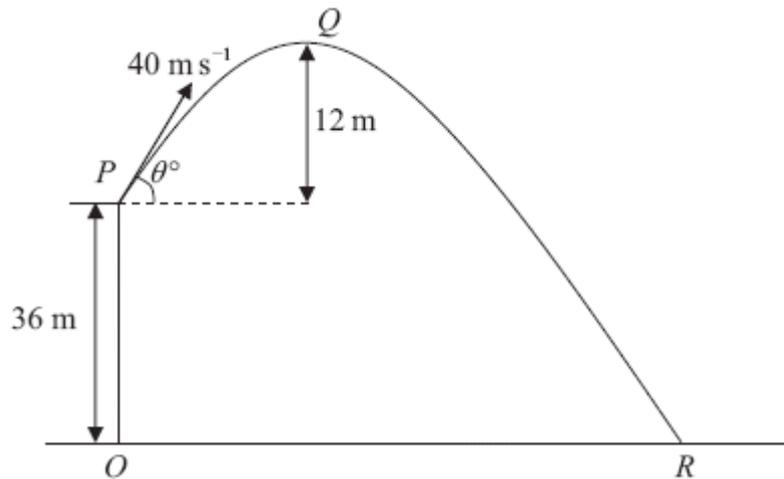

A ball is projected with speed 40 m s^{-1} from a point P on a cliff above horizontal ground. The point O on the ground is vertically below P and OP is 36 m. The ball is projected at an angle $\theta°$ to the horizontal. The point Q is the highest point of the path of the ball and is 12 m above the level of P. The ball moves freely under gravity and hits the ground at the point R, as shown in the diagram above. Find

(a)   the value of θ,

(3)

(b)   the distance OR,

(6)

(c)   the speed of the ball as it hits the ground at R.

(3)
(Total 12 marks)

2.   [*In this question* **i** *and* **j** *are unit vectors in a horizontal and upward vertical direction respectively*]

A particle P is projected from a fixed point O on horizontal ground with velocity $u(\mathbf{i} + c\mathbf{j})$ ms^{-1}, where c and u are positive constants. The particle moves freely under gravity until it strikes the ground at A, where it immediately comes to rest. Relative to O, the position vector of a point on the path of P is $(x\mathbf{i} + y\mathbf{j})$ m.

(a)   Show that

$$y = cx - \frac{4.9x^2}{u^2}.$$

(5)

Given that $u = 7$, $OA = R$ m and the maximum vertical height of P above the ground is H m,

(b)   using the result in part (a), or otherwise, find, in terms of c,
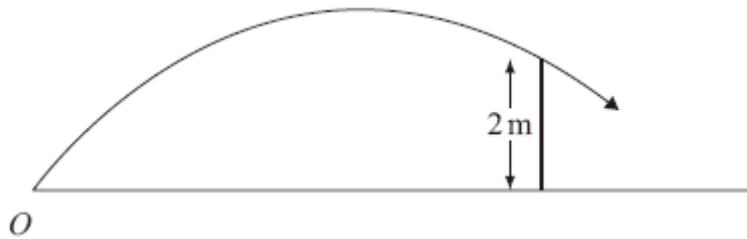
(i) R

(ii) H.

(6)

Given also that when P is at the point Q, the velocity of P is at right angles to its initial velocity,

(c)   find, in terms of c, the value of x at Q.

(6)
(Total 17 marks)

3.

A child playing cricket on horizontal ground hits the ball towards a fence 10 m away. The ball moves in a vertical plane which is perpendicular to the fence. The ball just passes over the top of the fence, which is 2 m above the ground, as shown in the diagram above.

The ball is modelled as a particle projected with initial speed u m s^{-1} from point O on the ground at an angle α to the ground.

(a)   By writing down expressions for the horizontal and vertical distances, from O of the ball t seconds after it was hit, show that

$$2 = 10 \tan \alpha - \frac{50g}{u^2 \cos^2 \alpha}.$$

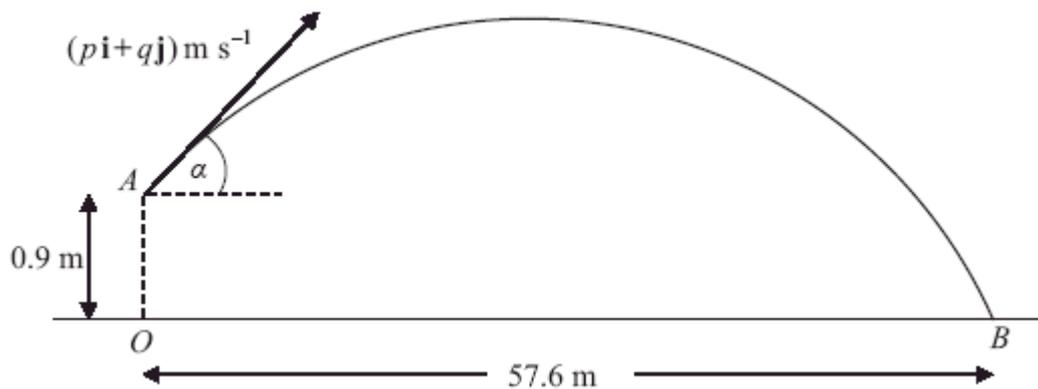**(6)**

Given that $\alpha = 45°$,

(b)   find the speed of the ball as it passes over the fence.

(6)
(Total 12 marks)

4.

A cricket ball is hit from a point A with velocity of $(p\mathbf{i} + q\mathbf{j})$ m s^{-1}, at an angle α above the horizontal. The unit vectors i and j are respectively horizontal and vertically upwards. The point A is 0.9 m vertically above the point O, which is on horizontal ground.

The ball takes 3 seconds to travel from A to B, where B is on the ground and $OB = 57.6$ m, as shown in the diagram above. By modelling the motion of the cricket ball as that of a particle moving freely under gravity,
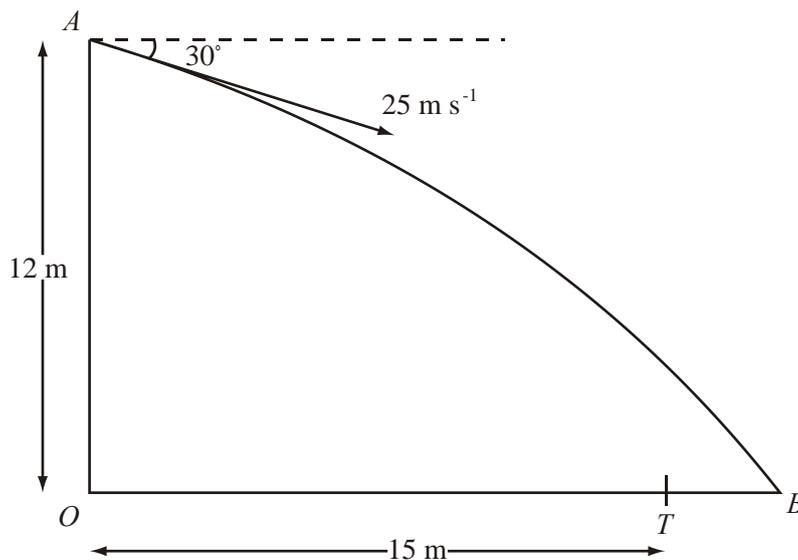
(a)   find the value of p,

(2)

(b)   show that $q = 14.4$,

(3)

(c)   find the initial speed of the cricket ball,

(2)

(d)   find the exact value of tan α.

(1)

(e)   Find the length of time for which the cricket ball is at least 4 m above the ground.

(6)

(f)   State an additional physical factor which may be taken into account in a refinement of the above model to make it more realistic.

(1)
(Total 15 marks)

5.

A ball is thrown from a point *A* at a target, which is on horizontal ground. The point *A* is 12 m above the point *O* on the ground. The ball is thrown from *A* with speed 25 m s^{-1} at an angle of 30° below the horizontal. The ball is modelled as a particle and the target as a point *T*. The distance *OT* is 15 m. The ball misses the target and hits the ground at the point *B*, where *OTB* is a straight line, as shown in the diagram above. Find

(a)   the time taken by the ball to travel from *A* to *B*,

(5)

(b)   the distance *TB*.

(4)

The point *X* is on the path of the ball vertically above *T*.

(c)   Find the speed of the ball at *X*.

(5)
(Total 14 marks)

6.
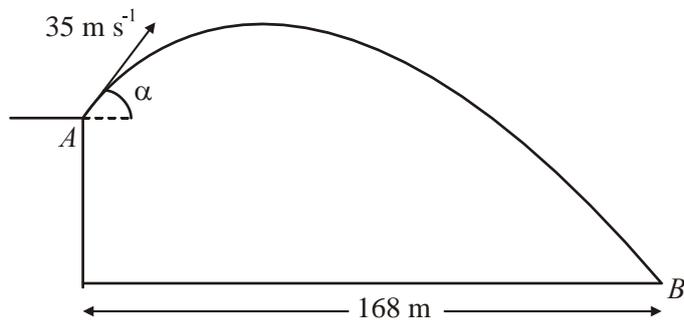
A golf ball P is projected with speed 35 m s^{-1} from a point A on a cliff above horizontal ground. The angle of projection is α to the horizontal, where $\tan \alpha = \dfrac{4}{3}$. The ball moves freely under gravity and hits the ground at the point B, as shown in the diagram above.

(a)   Find the greatest height of P above the level of A.

(3)

The horizontal distance from A to B is 168 m.

(b)   Find the height of A above the ground.

(6)

By considering energy, or otherwise,

(c)   find the speed of P as it hits the ground at B.

(3)
(Total 12 marks)

7.
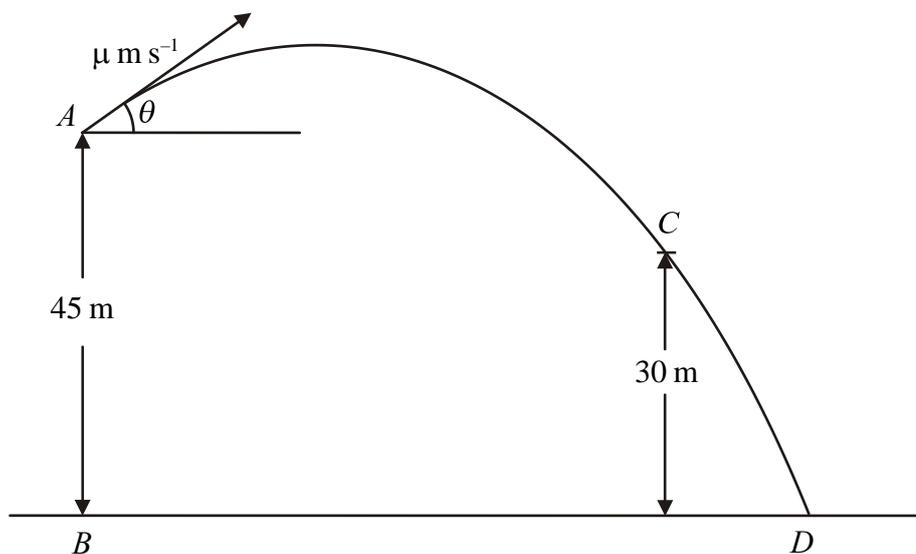
A particle P is projected from a point A with speed u m s^{-1} at an angle of elevation θ, where $\cos\theta = \dfrac{4}{5}$. The point B, on horizontal ground, is vertically below A and $AB = 45$ m.

After projection, P moves freely under gravity passing through a point C, 30 m above the ground, before striking the ground at the point D, as shown in Figure 3.

Given that P passes through C with speed 24.5 m s^{-1},

(a)   using conservation of energy, or otherwise, show that $u = 17.5$,

(4)

(b)   find the size of the angle which the velocity of P makes with the horizontal as P passes through C,

(3)

(c)   find the distance BD.

(7)
(Total 14 marks)

8.   A vertical cliff is 73.5 m high. Two stones A and B are projected simultaneously. Stone A is projected horizontally from the top of the cliff with speed 28 m s^{-1}. Stone B is projected from the bottom of the cliff with speed 35 m s^{-1} at an angle α above the horizontal. The stones move freely under gravity in the same vertical plane and collide in mid-air. By considering the horizontal motion of each stone,

   (a)   prove that $\cos\alpha = \dfrac{4}{5}$.

                                                                                                    (4)

   (b)   Find the time which elapses between the instant when the stones are projected and the instant when they collide.

                                                                                                    (4)
                                                                                          (Total 8 marks)

9.
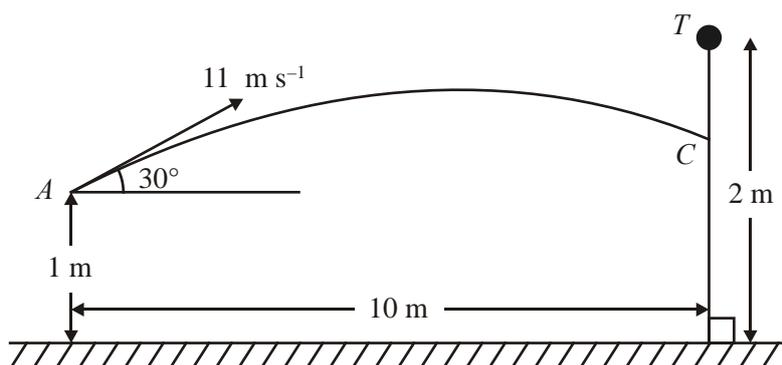
   The object of a game is to throw a ball B from a point A to hit a target T which is placed at the top of a vertical pole, as shown in the figure above. The point A is 1 m above horizontal ground and the height of the pole is 2 m. The pole is at a horizontal distance of 10 m from A. The ball B is projected from A with a speed of 11 m s^{-1} at an angle of elevation of 30°. The ball hits the pole at the point C. The ball B and the target T are modelled as particles.

   (a)   Calculate, to 2 decimal places, the time taken for B to move from A to C.

                                                                                                    (3)

   (b)   Show that C is approximately 0.63 m below T.

                                                                                                    (4)

The ball is thrown again from A. The speed of projection of B is increased to V m s^{-1}, the angle of elevation remaining 30°. This time B hits T.

(c)   Calculate the value of V.

(6)

(d)   Explain why, in practice, a range of values of V would result in B hitting the target.

(1)
(Total 14 marks)

10.   A darts player throws darts at a dart board which hangs vertically. The motion of a dart is modelled as that of a particle moving freely under gravity. The darts move in a vertical plane which is perpendicular to the plane of the dart board. A dart is thrown horizontally with speed 12.6 m s^{-1}. It hits the board at a point which is 10 cm below the level from which it was thrown.

(a)   Find the horizontal distance from the point where the dart was thrown to the dart board.

(4)

The darts player moves his position. He now throws a dart from a point which is at a horizontal distance of 2.5 m from the board. He throws the dart at an angle of elevation α to the horizontal, where $\tan \alpha = \frac{7}{24}$. This dart hits the board at a point which is at the same level as the point from which it was thrown.

(b)   Find the speed with which the dart is thrown.

(6)
(Total 10 marks)

11.
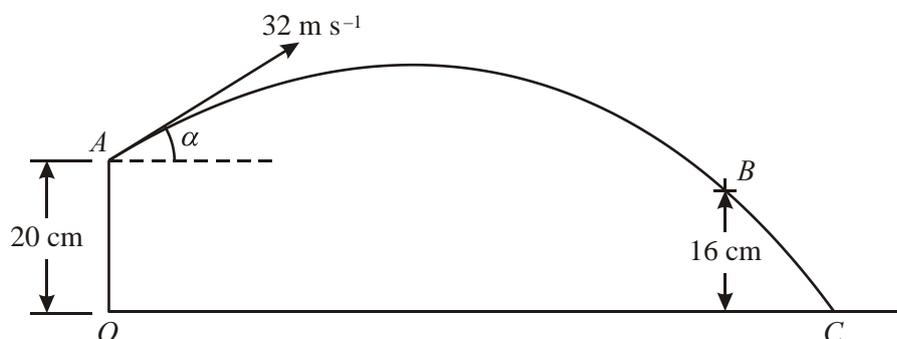
A particle *P* is projected from a point *A* with speed 32 m s^{-1} at an angle of elevation α, where sin α = $\frac{3}{5}$. The point *O* is on horizontal ground, with *O* vertically below *A* and *OA* = 20 m. The particle *P* moves freely under gravity and passes through a point *B*, which is 16 m above the ground, before reaching the ground at the point *C*, as shown in the diagram.

Calculate

(a)   the time of the flight from *A* to *C*,

(5)

(b)   the distance *OC*,

(3)

(c)   the speed of *P* at *B*,

(4)

(d)   the angle that the velocity of *P* at *B* makes with the horizontal.

(3)
(Total 15 marks)

12.
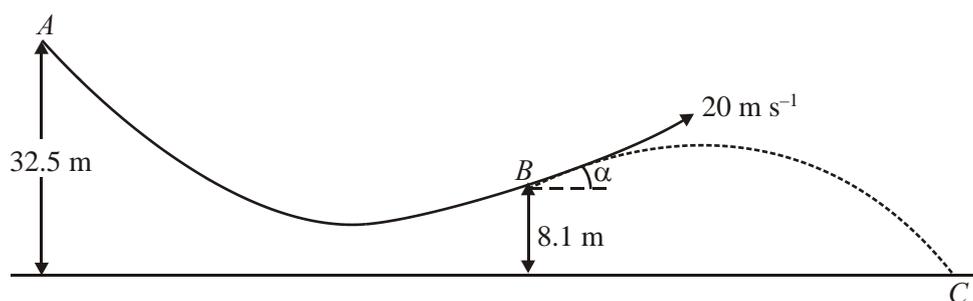
In a ski-jump competition, a skier of mass 80 kg moves from rest at a point *A* on a ski-slope. The skier's path is an arc *AB*. The starting point *A* of the slope is 32.5 m above horizontal ground. The end *B* of the slope is 8.1 m above the ground. When the skier reaches *B*, she is travelling at 20 m s^{-1}, and moving upwards at an angle α to the horizontal, where tan α = $\frac{3}{4}$, as shown in the diagram above. The distance along the slope from *A* to *B* is 60 m. The resistance to motion while she is on the slope is modelled as a force of constant magnitude *R* newtons. By using the work-energy principle,

(a)   find the value of *R*.

(5)

On reaching *B*, the skier then moves through the air and reaches the ground at the point *C*. The motion of the skier in moving from *B* to *C* is modelled as that of a particle moving freely under gravity.

(b)   Find the time for the skier to move from *B* to *C*.

(5)

(c)   Find the horizontal distance from *B* to *C*.

(2)

(d)   Find the speed of the skier immediately before she reaches *C*.

(5)
(Total 17 marks)

13.   A particle *P* is projected with velocity $(2u\mathbf{i} + 3u\mathbf{j})$ m s^{-1} from a point *O* on a horizontal plane, where **i** and **j** are horizontal and vertical unit vectors respectively. The particle *P* strikes the plane at the point *A* which is 735 m from *O*.

(a)   Show that *u* = 24.5.

(6)

(b)   Find the time of flight from *O* to *A*.

(2)

The particle *P* passes through a point *B* with speed 65 m s^{-1}.

(c)   Find the height of B above the horizontal plane.

(4)
(Total 12 marks)

14.
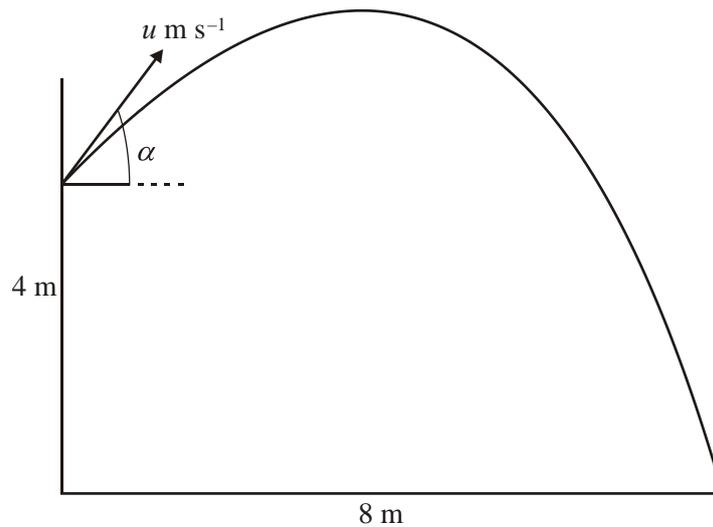
A ball is thrown from a point 4 m above horizontal ground.   The ball is projected at an angle
α above the horizontal, where $\tan \alpha = \frac{3}{4}$. The ball hits the ground at a point which is a
horizontal distance 8 m from its point of projection, as shown in the diagram above. The initial
speed of the ball is u m s^{-1} and the time of flight is T seconds.

(a)   Prove that $uT = 10$.

(2)

(b)   Find the value of u.

(5)

As the ball hits the ground, its direction of motion makes an angle ϕ with the horizontal.

(c)   Find $\tan \phi$.

(5)
(Total 12 marks)

15.
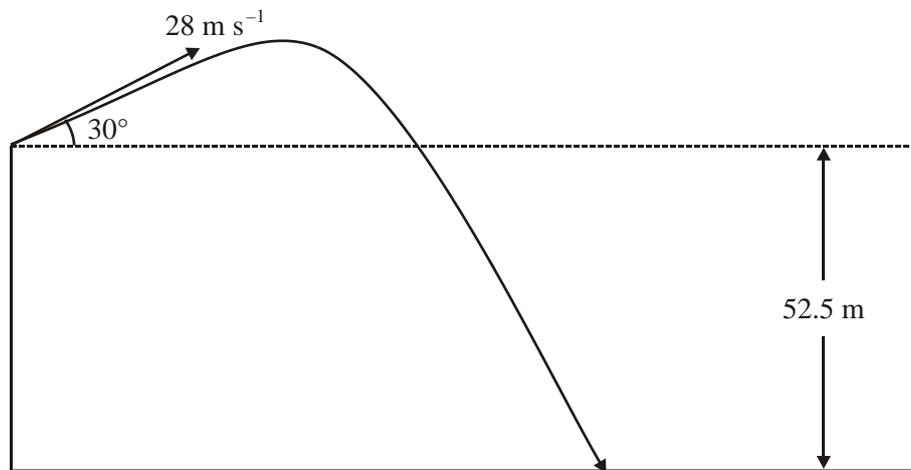
A shot is projected upwards from the top of a cliff with a velocity of 28 m s^{-1} at an angle of 30° above the horizontal. It strikes the ground 52.5 m vertically below the level of the point of projection, as shown in the diagram above. The motion of the shot is modelled as that of a particle moving freely under gravity.

Find, to 3 significant figures,

(a)   the horizontal distance from the point of projection at which the shot strikes the ground,

(8)

(b)   the speed of the shot as it strikes the ground.

(5)
(Total 13 marks)

1. (a)   Vertical motion: $v^2 = u^2 + 2as$ M1

$(40 \sin \theta^2) = 2 \times g \times 12$ A1

$(\sin \theta)^2 = \dfrac{2 \times g \times 12}{40^2}$

$\theta = 22.54 = 22.5°$ (accept 23)   A1   3

(b)   Vert motion $P \to R$: $s = ut + \frac{1}{2}at^2$

$-36 = 40 \sin \theta t - \dfrac{g}{2}t^2$ M1

$\dfrac{g}{2}t^2 - 40 \sin \theta t - 36 = 0$ A1 A1

$t = \dfrac{40 \sin 22.54 + \sqrt{(40 \sin 22.54)^2 + 4 \times 4.9 \times 36}}{9.8}$

$t = 4.694...$ A1

Horizontal P to R: $s = 40 \cos \theta t$ M1

$= 173$ m (or 170 m)   A1   6

(c)   Using Energy:

$\dfrac{1}{2}mv^2 - \dfrac{1}{2}m \times 40^2 = m \times g \times 36$ M1 A1

$v^2 = 2\left(9.8 \times 36 + \frac{1}{2} \times 40^2\right)$

$v = 48.0.....$

$v = 48$ m s^{-1} (accept 48.0)   A1   3

[12]

2. (a) $x = ut$ B1

$y = cut - 4.9t^2$ M1 A1

eliminating t and simplifying to give $y = cx - \dfrac{4.9x^2}{u^2}$ **   DM1 A1   5

(b) (i) $0 = cx - \dfrac{4.9x^2}{u^2}$ M1

$0 = x\left(c - \dfrac{4.9x}{u^2}\right) \Rightarrow R = \dfrac{u^2 c}{4.9} = 10c$ M1 A1

(ii)   When $x = 5c, y = H$ M1

$$= 5c^2 - \frac{(5c)^2}{10} = 2.5c^2 \qquad\qquad \text{M1 A1} \qquad 6$$

(c) $$\frac{dy}{dx} = c - \frac{9.8x}{u^2} = c - \frac{x}{5} \qquad\qquad \text{M1 A1}$$

When $x = 0$, $\dfrac{dy}{dx} = c$ $\qquad\qquad$ B1

So, $c - \dfrac{x}{5} = \dfrac{-1}{c}$ $\qquad\qquad$ DM1 A1

$$x = 5\left(c + \frac{1}{c} \right) \qquad\qquad \text{A1} \qquad 6$$

<u>Alternative</u>
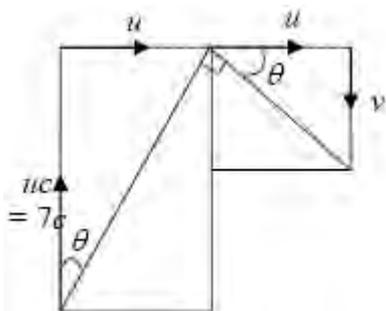
$$\tan\theta = \frac{u}{cu} = \frac{1}{c} = \frac{v}{u} \qquad\qquad \text{B1}$$

$$\Rightarrow v = \frac{u}{c} = \frac{7}{c} \qquad\qquad \text{M1 A1}$$

$$v = u + at; \qquad -\frac{7}{c} = 7c - 9.8t \qquad\qquad \text{M1}$$

$$t = \frac{7}{9.8}\left(c + \frac{1}{c} \right) \qquad\qquad \text{A1}$$

$$x = ut = 7t; \qquad x = 5\left(c + \frac{1}{c} \right) \qquad\qquad \text{A1}$$

[17]

3.   (a) $\rightarrow x = u \cos \alpha t = 10$ M1A1

           $\uparrow \quad y = u \sin \alpha t - \dfrac{1}{2} g t^2 = 2$ M1A1

           $\Rightarrow t = \dfrac{10}{u \cos \alpha}$

           $2 = u \sin \alpha \times \dfrac{10}{u \cos \alpha} - \dfrac{g}{2} \times \dfrac{100}{u^2 \cos^2 \alpha}$ M1

           $= 10 \tan \alpha - \dfrac{50g}{u^2 \cos^2 \alpha}$ (given answer)   A1   6

    (b) $2 = 10 \times 1 - \dfrac{100g \times 2}{2u^2 \times 1}$ M1A1

           $u^2 = \dfrac{100g}{8}, u = \sqrt{\dfrac{100g}{8}} = 11.1 \left(\mathrm{m \, s}^{-1} \right)$ A1

           $\dfrac{1}{2} m u^2 = m \times 9.8 \times 2 + \dfrac{1}{2} m v^2$ M1A1

           $v = 9.1 ms^{-1}$ A1   6

                                                               [12]

4.   (a)   Horizontal distance: $57.6 = p \times 3$ M1

                                 $p = 19.2$ A1   2

    (b)   Use $s = ut + \dfrac{1}{2} a t^2$ for vertical displacement.   M1

           $-0.9 = q \times 3 - \dfrac{1}{2} g \times 3^2$ A1

           $-0.9 = 3q - \dfrac{9g}{2} = 3q - 44.1$

           $q = \dfrac{43.2}{3} = 14.4$ * **AG** *   A1 cso   3

    (c)   initial speed $\sqrt{p^2 + 14.4^2}$ (with their p)   M1

                 $= \sqrt{576} = \underline{24} \, (\mathrm{m \, s}^{-1})$ A1 cao   2

(d) $\tan\alpha = \dfrac{14.4}{p}\ (=\dfrac{3}{4})$ (with their p)   B1   1

(e)   When the ball is 4 m above ground:

$3.1 = ut + \dfrac{1}{2}at^2$ used   M1

$3.1 = 14.4t - \dfrac{1}{2}gt^2$ o.e $(4.9t^2 - 14.4t + 3.1 = 0)$ A1

$\Rightarrow t = \dfrac{14.4 \pm \sqrt{(14.4)^2 - 4(4.9)(3.1)}}{2(4.9)}$ seen or implied   M1

$t = \dfrac{14.4 \pm \sqrt{146.6}}{9.8} = 0.023389...$or $2.70488...$awrt 0.23 and 2.7   A1

duration = 2.70488... – 0.23389...   M1
          = 2.47 or 2.5 (seconds)   A1   6

or

M1A1M1 as above

$t = \dfrac{14.4 \pm \sqrt{146.6}}{9.8}$ A1

Duration $2 \times \dfrac{\sqrt{146.6}}{9.8}$ o.e.   M1

       = 2.47 or 2.5 (seconds)   A1   6

(f)   Eg. : Variable 'g', Air resistance, Speed of wind, Swing of ball,
             The ball is not a particle.   B1   1

                                                                              [15]

5.   (a) $(\downarrow)\ u_y = 25 \sin30° (= 12.5)$ B1
             $12 = 12.5t + 4.9t^2$ – 1 each error   M1A2(1,0)
             Leading to $t = 0.743, 0.74$ A1   5

       (b) $(\rightarrow)\ u_x = 25 \cos 30° \left(= \dfrac{25\sqrt{3}}{2} \approx 21.65 \right)$ B1

             $OB = 25 \cos 30° \times t\ (\approx 16.094\ 58)$ ft their (a)   M1A1ft
             $TB \approx 1.1\ (m)$ awrt 1.09   A1   4

(c)   ($\rightarrow$) $15 = u_x \times t \Rightarrow t = \dfrac{15}{u_x}\ (=\dfrac{2\sqrt{3}}{5}\ \approx 0.693$ or $0.69)$ M1A1

either ($\downarrow$) $v_y = 12.5 + 9.8t\ (\approx 19.2896)$ M1

$V^2 = u_x^2 + v_y^2\ (\approx 840.840)$

$V \approx 29$ (m s^{-1}), 29.0   M1A1   5

or   ($\downarrow$) $s_y = 12.5t + 4.9t^2\ (\approx 11.0)$ M1

$\dfrac{1}{2}m \times 25^2 + mg \times s_y = \dfrac{1}{2}mv^2$

$V \approx 29$ (m s^{-1}), 29.0   M1A1

[14]

6.   (a) $0 = (35 \sin \alpha)^2 - 2gh$ M1A1

$h = \underline{40\ m}$ A1   3

(b) $x = 168 \Rightarrow 168 = 35 \cos \alpha.t\ (\Rightarrow t = 8\text{s})$ M1A1

At $t = 8$, $y = 35\sin\alpha \times t - \dfrac{1}{2}gt^2\ (= 28.8 - \frac{1}{2}.g.8^2 = -89.6\ \text{m})$ M1A1

Hence height of $A = \underline{89.6\ m}$ or 90 m   DM1A1   6

(c) $\frac{1}{2}mv^2 = \frac{1}{2}.m.35^2 + mg.89.6$ M1A1

$\Rightarrow v = \underline{54.6\ \text{or}\ 55\ \text{m s}^{-1}}$ A1   3

M1   Use of $v^2 = u^2 + 2as$, or possibly a 2 stage method using $v = u + at$
     and $s = ut + \dfrac{1}{2}at^2$

A1   Correct expression. Alternatives need a complete method leading to an
     equation in *h* only.

A1   40(m) No more than 2sf due to use of g.

M1   Use of $x = u\cos\alpha \cdot t$ to find *t*.

A1 $168 = 35 \times$ their $\cos\alpha \times t$

M1   Use of $s = ut + \dfrac{1}{2}at^2$ to find vertical distance for their *t*. (*AB* or top to *B*)

A1 $y = 35\sin\alpha \times t - \dfrac{1}{2}gt^2$ (*u*, *t* consistent)

DM1 This mark dependent of the previous 2 M marks. Complete method for *AB*.
     Eliminate *t* and solve for *s*.

A1   cso.

(NB some candidates will make heavy weather of this, working from *A* to max
height (40m) and then down again to *B* (129.6m))

OR: Using $y = x\tan\alpha - \dfrac{gx^2\sec^2\alpha}{2u^2}$
     M1 formula used (condone sign error)
     A1 *x*, *u* substituted correctly
     M1 α terms substituted correctly.
     A1 fully correct formula
     M1, A1 as above

M1   Conservation of energy: change in KE = change in GPE.
     All terms present. One side correct (follow their *h*).
     (will probably work *A* to *B*, but could work top to *B*).

A1   Correct expression (follow their *h*)

A1   54.6 or 55 (m/s)

OR: M1   horizontal and vertical components found and combined
          using Pythagoras
          $v_x = 21$
          $v_y = 28 - 9.8 \times 8\ (-50.4)$
     A1 v_x and v_y expressions correct (as above). Follow their *h*, *t*.
     A1   54.6 or 55

*NB Penalty for inappropriate rounding after use of g only applies once
per question.*

[12]

7.   (a)   Energy $\frac{1}{2}m(24.5^2 - u^2) = mg \times 15$ M1A1=A1

$u^2 = 24.5^2 - 30g = 306.25$

$u = \sqrt{306.25} = 17.5$ *   cso   A1   4

Alternative

$\rightarrow u_x = u \cos \theta = 0.8u, \uparrow u_y = u \sin \theta = 0.6u$

$v_y^2 = 0.36u^2 + 2 \times 9.8 \times 15 = 0.36u^2 + 294$

$24.5^2 = u_x^2 + v_y^2 = 0.64u^2, +0.36u^2 + 294$ M1 A1,A1

$u^2 = 306.25 \Rightarrow u = 17.5$ *   cso   A1   4

(b) $\rightarrow u_x = u \cos \theta = 17.5 \times 0.8 = 14$ B1

$\psi = \arccos \frac{14}{24.5} \approx 55°$ accept 55.2°   M1A1   3

(0.96 rads; or 0.963 rads)

Alternative

$\rightarrow u_x = u \cos \theta = 17.5 \times 0.8 = 14$ B1

$\uparrow v_y^2 = u^2 \sin^2 \theta + 2 \times 9.8 \times 15 = 404.25$

$\psi = \arctan \frac{\sqrt{404.25}}{14} \approx 55°$ accept 55.2°   M1A1   3

(c) $\uparrow u_y = u \sin \theta = 17.5 \times 0.6 = 10.5$ B1

$s = ut + \frac{1}{2}at^2 \Rightarrow -45 = 10.5t - 4.9t^2$ M1 A1

leading to $t = 4.3$. awrt $t = 4.3$ or $t = 4\frac{2}{7}$ A1

$\rightarrow BD = 14 \times 4\frac{2}{7}$ $(14 \times t)$ ft their t M1 A1ft

$= 60$ (m) only   A1   7

Alternative

Use of $y = x \tan \theta - \frac{g \sec^2 \vartheta}{2u^2}x^2$ M1

$-45 = \frac{3}{4}x, -\frac{g}{2 \times 17.5^2} \times \frac{25}{16}x^2$ B1,A1

$x^2 - 30x - 1800 = 0$ o.e.   A1

Factors or quadratic formula   M1 A1ft

$BD = 60$ (m)   A1

[14]

8. (a) $x_A = 28t$ $x_B = 35 \cos \alpha\, t$ B1 B1

Meet $\Rightarrow 28t = 35 \cos \alpha\, t \Rightarrow \cos \alpha = 28 / 35 = 4 / 5$ *   M1 A1   4

(b) $y_A = 73.5 - \frac{1}{2} gt^2$ $y_B = 2lt - \frac{1}{2} gt^2$ B1 B1

Meet $\Rightarrow 73.5 = 21t \Rightarrow t = \underline{3.5\ \text{s}}$ M1 A1   4
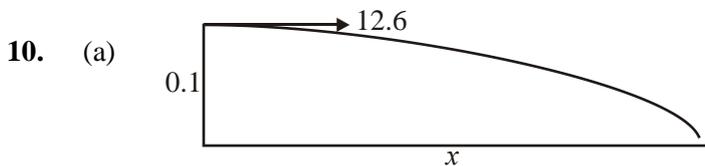
[8]

9. (a) $u_x = \ 11 \cos 30°$ B1

$\rightarrow 11 \cos 30° \times t = 10 \quad \Rightarrow \quad t = 1.05 \quad$ (s)   cao   M1 A1   3

(b) $s = \underline{11 \cos 30°} \times t - 4.9t^2 \approx 0.37$ B1 M1 A1

$(2 - 1) - 0.37 = 0.63 \quad$ (m)   A1   4
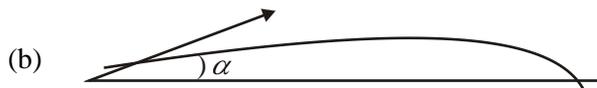
(c) $V \cos 30° \times t = 10 \qquad \left(t = \dfrac{10}{V \cos 30°} \right)$ M1 A1

$s = V \sin 30° \times \dfrac{10}{V \cos 30°} - \dfrac{4.9 \times 100}{V^2 \cos^2 \theta} = 1$ M1 A1

$V^2 = 136.86$ M1
$V \approx 12$ accept 11.7   A1   6

(d) B and/or T are not particles   B1   1

(They have extension giving a range of answers)

[14]

10. (a)

$\rightarrow 12.6t = x$ B1
$\downarrow 0.1 = 4.9t^2$ B1
$\Rightarrow 0.1 = 4.9 \times \dfrac{x^2}{12.6}$ M1
$\Rightarrow x = 1.8\ \text{m}$ A1   4

(b)

$\rightarrow u \cos \alpha . t = 2.5$ M1 A1

$\uparrow u \sin \alpha . t = \dfrac{1}{2} g t^2$ M1 A1

$u . \dfrac{24}{25} t = 2.5$

$u . \dfrac{7}{25} = 4.9 . \dfrac{2.5 . 25}{24 u}$

$u^2 = \dfrac{4.9 \times 2.5 \times 25^2}{7 \times 24}$

$\Rightarrow u \approx 6.75$ or 6.8 m s^{-1} M1 A1   6

                                                                                    [10]

11.   (a) $\uparrow \quad u_y = 32 \times \dfrac{3}{5} \ (= 19.2)$ B1

$-20 = 19.2t - 4.9t^2$ M1 A2 (1, 0)

                    –1 each error

$t \approx 4.8$ or 4.77 (s)   A1   5

(b) $\rightarrow \quad u_x = 32 \times \dfrac{4}{5} \ (= 25.6)$ B1

$d = 25.6 \times 4.77 \ldots$ M1

≈ 120 or 122 (m)   A1   3

(c) $\uparrow \quad v_y^2 = 19.2^2 + 2 \times 9.8 \times 4 \ [v_y^2 = 447.04, v_y \approx 21.14]$ M1

$V^2 = 447.04 + 25.6^2$ M1 A1

$V = 33$ or 33.2 (ms^{-1})   A1   4

(d) $\tan \theta = \dfrac{21.14}{25.6} \ \left(\text{or } \cos \theta = \dfrac{25.6}{33.2}, \ldots \right)$ M1 A1ft

                    ft their components
                    or resultant

$\theta \approx 40°$ or $39.6°$ A1   3

                                                                                    [15]

Alternative for (c)

$$\frac{1}{2}m(V^2 - 32^2) = mg \times 4 \qquad\qquad \text{M1 A1}$$

$$V^2 = 1102.4 \qquad\qquad \text{M1}$$

$$V = 33 \text{ or } 33.2 \text{ (ms}^{-1}) \qquad\qquad \text{A1} \qquad 4$$

There is a maximum penalty of one mark per question for not rounding to appropriate accuracy.

12. (a) Work-Energy $R \times 60 = 80 \times 9.8 \times 24.4 - \dfrac{1}{2} \times 80 \times 20^2$ M1 A2(1, 0)

   $(= 19129.6 - 16000 = 3129.6)$

   $R = 52$ (N)   M1 A1   5

        accept 52.2

   (b) $-8.1 = 20 \sin \alpha \times t - \dfrac{1}{2}gt^2$ M1 A2(1, 0)

   $4.9t^2 - 12t - 8.1 = 0$ M1

   $t = 3$ (s)   A1   5

   (c) $20\cos\alpha \times 3 = 16 \times 3 = 48$ (m)   M1 A1ft   2

        ft their t

   (d)   Energy $\dfrac{1}{2}mv^2 - \dfrac{1}{2}m \times 20^2 = m \times 9.8 \times 8.1$ M1 A2(1, 0)

   $v = \sqrt{(558.56)} \approx 24$ (ms^{-1})   M1 A1   5

        accept 23.6

   [17]

Alternative to (d)

$\uparrow v_y = 12 - 3g = -17.4$ M1 A1
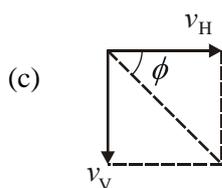
$\rightarrow v_x = 16$ A1

$v = \sqrt{(17.4^2 + 16^2)} \approx 24$ (ms^{-1})   M1 A1   5

        accept 23.6

13. (a) $2ut = 735$ M1 A1

        $0 = 3ut - \dfrac{1}{2} gt^2$ M1 A1

        eliminating t **dep.** M1

        $u = 24.5$ *   A1   6

      (b) $t = \dfrac{735}{49} = 15$ M1 A1   2

      (c)   Initially: $v^2 = (2u)^2 + (3u)^2$ (7803.25)   M1

           $\dfrac{1}{2} mv^2 - \dfrac{1}{2} m\, 65^2 = mgh$ M1 A1

           $h = 180$ m (183 m)   A1   4

     OR $v_y^2 = 65^2 - (2u)^2$ (1824)   M1

           $v_y^2 = (3u)^2 - 2gh$ M1 A1

           $h = 180$ m (183 m)   A1   (4)

                                                                             [12]

14. (a) $(\rightarrow)$: $u \cos \alpha \times T = 8$

           $u \times \frac{4}{5} \times T = 8$ M1

           $uT = 10$ (*)   A1   2

      (b) $(\uparrow)$: $-4 = u \sin \alpha\, T - \frac{1}{2} gT^2$ M1 A1

           $-4 = u \times \dfrac{3}{5} \left(\dfrac{10}{u}\right) - \dfrac{1}{2} \times 9.8 \left(\dfrac{10}{u}\right)^2$ M1

           $u = 7$ M1 A1   7

(c)

$v_H = u \cos \alpha = \frac{28}{5}$ B1 ft

$v_V^2 = (-u \sin \alpha)^2 + 2g \times 4$ M1

$\Rightarrow v_V = 9.8 \ (= \frac{49}{5})$ A1 ft

$\tan \phi = \dfrac{49/5}{28/5} = \dfrac{7}{4}$ M1 A1 cao 5

  [12]

15. (a) $(\uparrow)$: $-52.5 = 14t - \frac{1}{2} \times 9.8t^2$ M1 A2

$7t^2 - 20t - 75 = 0$

$(7t + 15)(t - 5) = 0$

$t = 5$ (or $t = -\frac{15}{7}$) M1 A1 A1

$(\rightarrow)$: $S = 28 \cos 30° \times 5$ M1

$= 70\sqrt{3} = 121$ m (3 s.f.) A1 8

(b) $v_{\text{horizontal}}$: $28 \cos 30° = 14\sqrt{3}$

v_{vertical}: $28 \sin 30° - 5g = -35$ M1 A1

$\therefore$ speed $= \sqrt{((14\sqrt{3})^2 + 35^2)} = \sqrt{1813} = 42.6$ m s^{-1} M1 A1 5

OR

KE gain = PE loss

$\frac{1}{2} m (v^2 - 28^2) = mg \times 52.5$ M1 A2

$\Rightarrow v = \sqrt{1813} = 42.6$ m s^{-1} M1 A1 5

  [13]

1. There were many confident solutions to this question, but over specification of answers
    following the use of $g = 9.8$ was a common problem, causing many candidates to throw away a
    mark.

    In part (a) many students used correct methods for calculating the angle, although 22.54 rather
    than 22.5 was common. Very few went wrong here, though some took longer routes than
    necessary, failing to spot that they could use $v^2 = u^2 + 2as$ to obtain the angle in one step, and
    there were a few who attempted to use distances to find the angle.

    In part (b) the quickest method here was to use a displacement of –36 in an equation to find time
    and then use this time in a horizontal equation to find displacement. Some took this in two
    stages: time to the highest point and then time to the bottom. It was pleasing to find very few
    resolution and both methods were used correctly in many solutions. The most common error
    was to consider only part of the flight and then use an incorrect time to find the horizontal
    displacement. The over-specified answer 173.4 rather than 173 was common.

    In part (c) those candidates who used conservation of energy were usually successful but the
    most common method was to find horizontal and vertical components of velocity and hence
    find the speed using Pythagoras' Theorem. Unfortunately, those who used this method often
    found only the vertical component of the velocity and lost all the marks here.

2. (a) Many correct solutions were seen, with the majority of candidates clearly familiar with
        the method for deriving the equation of the parabolic path. However some candidates
        substituted into all the suvat equations and were clearly struggling to find a way forward.

    (b) Those candidates who were not able to complete (a) started afresh at this point, and were
        often successful in earning marks in this part. Most candidates used the given formula to
        find R having identified this as the value of x when y = 0. Some candidates used the fact

        that maximum height occurs when $x = \dfrac{R}{2}$, but many preferred to work with the initial

        information and to use $v^2 = u^2 + 2as$ to find H.

    (c) Only a few candidates made a concerted attempt at this part with a correct solution a
        rarity. Most attempts used a vector approach rather than calculus. Several candidates
        demonstrated an understanding of perpendicular vectors, and the partially correct velocity

        $\begin{pmatrix} cu \\ -u \end{pmatrix}$ in place of the correct answer $\begin{pmatrix} u \\ \dfrac{-u}{c} \end{pmatrix}$ was quite common. Diagrams were rarely

        seen, possibly accounting for the common incorrect answer $\begin{pmatrix} u \\ -cu \end{pmatrix}$. Confusion

        between distances and velocities often marred any work beyond this stage.

3.   Part (a) was answered well by many candidates. The structure of the question, explicitly requiring the horizontal and vertical components as a starting point, led to many more successful answers than might otherwise have been the case. Most achieved the method marks successfully for both components but several had a sign error for the vertical component or some sin/cos confusion. As the answer was given, most candidates managed to recover sufficiently from initial errors to gain full marks. There were a few cases when a candidate had an incorrect expression but then wrote down the correct result! Some lost the final mark because another step was needed to show the given result.

In part (b) many candidates managed to use the given formula, allowing them to find u correctly, even if they had made an error in part (a). However, a significant number of candidates made errors in manipulating the given expression arriving at an incorrect value for u. A few candidates did not see that parts (a) and (b) were connected and started again with horizontal and vertical components. Others did not realise they could find *u* by substituting into the given equation. It was common for candidates to lose the last three marks. In finding u some thought they had already found the speed as it passed over the fence and proceeded no further.

A few candidates chose to use the more simple method of work-energy for the final part. These candidates usually scored full marks. The most common approach was to try to combine horizontal and vertical components of the final velocity. Some found only one component of the speed (usually the vertical one) and gave this as their answer. Others used u unresolved in either the vertical or horizontal direction and then combined to find v incorrectly. There were a significant number of responses making inappropriate use of $v^2 = u^2 + 2as$ to gain 9.13 incorrectly.

A few candidates scored no marks in part (b) because they made no attempt to find *u*.

4.   Many candidates scored well in this question, with parts (a) to (d) generally answered correctly. However, a few candidates were confused right at the start with the vector form of the initial velocity and tried to bring resolution into the problem and so failed to find *p* and *q*.

Some candidates were initially confused about the direction of *p* and *q* – it was common to see attempts at parts (a) and (b) relabelled when a candidate discovered their error. In (b) there was often insufficient evidence that the given answer had been reached correctly – essential steps in the working were omitted. Some candidates used long drawn out trigonometric methods to find

tan *α* in part (d), often finding cosα☐and sin *α* before finally reaching $\dfrac{3}{4}$.

In part (e) most candidates used $s = ut + \dfrac{1}{2}at^2$ from the point of projection but there were a number of other possible methods which were also successful. It was evident that some candidates are relying on the use of calculators to solve quadratic equations. When the initial quadratic equation was incorrect, marks were often lost as a result of failing to show sufficient evidence of use of an appropriate method. It was common to see 4 used in place of 3.1 in the initial equation. Candidates using alternative approaches often got lost in the complexities of the logic of what they were trying to do.

The responses in part (f) showed that virtually all candidates could find (at least) one physical factor which could also be taken into account although a few miss-worded their answers to imply the opposite. For example, many suggested "air resistance", but it was also common to see "no air resistance".

5. This was a well answered question with almost all candidates working in appropriate vertical and horizontal directions. However, some candidates were confused by the fact that the ball was projected below the horizontal – many of these assuming that it was projected horizontally and then followed the path illustrated.

    (a) Most candidates used $s = ut + \frac{1}{2}at^2$ to produce a quadratic equation albeit often with a sign error. When marks were lost this was usually because of difficulties with positive and negative vertical terms. The alternative, longer method, of finding v then t was not infrequent.

    (b) The method was clearly understood but the numbers involved were very sensitive to rounding errors. Too many candidates over-rounded their value for t and obtained the inaccurate value of 1.02.

    (c) It was pleasing to see that most candidates adopted the correct approach of finding t then vy but some failed to then combine v_x and v_y to obtain the speed. Again, sign errors with the acceleration resulted in errors in v_y being less than the original vertical component, which one would have hoped might ring an alarm bell.
Few candidates used the energy method and those that did often used the incorrect vertical displacement.

6.   (a) There were several instances of a possible misread of $\tan \alpha = \dfrac{3}{4}$ in this question, although it was not always possible to tell whether the error was a misread or use of an incorrect expression $\tan \alpha = \dfrac{\cos \alpha}{\sin \alpha}$. Most candidates used the formula as $v^2 = u^2 + 2as$ to find the maximum height. This was often found correctly, but a common error was to forget to square their value for u. Some candidates made the task more difficult than necessary by adopting a method with two, or more, stages.

    (b) Having found the time to travel a horizontal distance of 168m it is possible to find the vertical distance in one step, but many candidates elected either to find the time from the highest point to the ground, or to find the time from when the ball returns to the level of A until it reaches B. Candidates choosing one of these longer alternatives did not always match up their value for time taken with an appropriate value for the initial vertical speed. Having reached a value for vertical distance there was then some confusion about whether or not to add or subtract their answer from part (a).

    (c) Whether using energy or an alternative approach, the most common error in this part of the question was to concentrate on the vertical speed and to ignore or omit the horizontal component. This resulted in many candidates scoring no marks here.

7. This proved to be the most challenging question. Although it looked as though some candidates had run out of time to complete this question satisfactorily, others had sufficient time to make multiple attempts. Some candidates failed to appreciate the nature of velocity direction and tried to use equations for constant acceleration without taking direction into consideration. The most successful way to answer the first part was to use the method suggested – conservation of energy. In (b) some failed to appreciate that the velocity at C was at an angle (not necessarily equal to ◆ and that the horizontal velocity at A was constant throughout. Few candidates answered this concisely. The greatest difficulties were encountered in (c). A significant number of candidates failed to appreciate the use of displacement in constant acceleration equations and broke up the problem, quite unnecessarily, into sections where the particle was travelling up and then travelling down, making the solution of the problem much more difficult than it needed to be. There are several possible approaches to this question, some of them producing pleasingly concise solutions.

8. The first part was reasonably well done although some candidates failed to appreciate that it was a 4 mark proof and therefore required a full explanation – those that simply equated the horizontal velocity components only scored two of the four marks. Part (b) was more discriminating and many didn't appreciate that they needed two vertical distance-time equations and of those that did, a significant number were unable to combine them correctly.

9. Parts (a) and (b) were usually correct although some candidates made things more difficult for themselves by splitting the motion into "up and down" etc but they still usually got there in the end. The third part was more challenging but most candidates were able to pick up some marks although the algebraic manipulation required to solve their equations for V defeated some of the weaker candidates. Part (d) proved to be a good discriminator, with most candidates simply trotting out the "usual" air resistance without really thinking about the situation.

10. Part (a) was generally well done although a significant minority used 10 cm instead of 0.1 m and were not put off by the fact that this meant that the darts player was throwing darts at a board which was 18 m away! The second part proved to be more challenging. Most tried to work from 1st principles and set up two equations and eliminate t but a few successfully used quoted formulae for the range or the equation of the path. Candidates are reminded that in this case if they misquote formulae they will lose all of the marks.

11. As has been noted in previous examinations, those who attempted part (a) in one go, obtaining an equation equivalent to $4.9t^2 - 19.2t - 20 = 0$ were usually successful and, although there were some slips in sign, these were not common. Those who broke the time up into sections, for example, from *A* to the maximum height and from the maximum height to *C*, produced completely correct solutions less frequently. More stages in a calculation give more scope for error. Such methods are also, not infrequently, given in an incomplete form. Part (b) was well done and an incorrect answer in part (a) lost only one mark here. The most popular method in part (c) was to consider the components of the velocity at *B*. The alternative method using conservation of energy was rarely seen. It was not uncommon for candidates to find the vertical component at *B* and stop. This often did seem to be an error in understanding what was asked for in this part of the question, as such candidates often interpreted their answer correctly and completed part (d) successfully. An inefficient method of solution to part (c), not infrequently seen, was to find the time of flight from *A* to *B* and then to find the vertical component of the velocity. This calculation is awkward and carries a greater risk of sign errors than using $v^2 = u^2 + 2as$. Part (d) was well done although, as noted above, errors due to premature approximation were often seen.

12. Part (a) of this question proved to be the most testing and the work-energy principle was not understood by many candidates. It was not unusual to see kinetic energy omitted entirely and, when it was included, it often had the wrong sign. Some candidates did not know that work was force multiplied by distance and force multiplied by velocity was not infrequently seen. Those who attempted part (b) in one go, obtaining an equation equivalent to $4.9t^2 - 12t - 8.1 = 0$ were usually successful, although there were, as expected, some slips in sign. Those who broke the time up into sections, for example, from *B* to the maximum height and from the maximum height to *C*, produced completely correct solutions much less frequently although it is not clear if this is because of the intrinsic difficulty of the method or if it was because this method was favoured by weaker candidates. Any answer in part (b) was followed through in part (c) and the majority of the candidates gained the two marks here. The most popular method in part (d) was to consider the components of the velocity at *C*. A few having found the vertical component at *C* stopped but this part of the question was often completed successfully. The alternative method using conservation of energy was rarely seen but, when seen, it was usually correct.

13. Many candidates coped well with parts (a) and (b). Those who provided correct solutions to part (c) usually did so using vertical components of velocity rather than energy. A common error in (c) was to treat 65 as the vertical component of velocity, and use it in $v^2 = u^2 + 2as$. This gave the correct answer but received no credit. A number of candidates interchanged components and then proceeded well. A major source of error came from candidates used to using *u* and an angle for initial conditions who often spent a lot of time and space converting vector form to non-vector form, rarely with success.

Some quoted a formula for the range and confused *u* in the formula with *u* in question. Comparatively few completed part (c) and it was therefore very disappointing that some who managed it lost the final mark by giving the answer to 4 or 5 sig figs. They are generally getting better at this but still need to be reminded.

14.   (a)   There were many correct solutions to this part although the weaker candidates struggled
              generally with the whole of this question.

      (b)   This was more demanding and discriminated well at the top end. Common errors were
              signs and failing to resolve *u*. Those who tried to split the motion were rarely successful.

      (c)   The method here was generally well known with few candidates using distances in their
              tan expression.

15.   No Report available for this question.